RIGHT TRIANGLE TRIGONOMETRY:
(November 14 - December 16)

In this unit students will explore the relationships that exist between sides and angles of right triangles, build upon their previous knowledge of similar triangles and of the Pythagorean Theorem to determine the side length ratios in special right triangles. Students will also understand the conceptual basis for the functional ratios sine and cosine, explore how to use trigonometric ratios to solve problems and how the values of these trigonometric functions relate in complementary angles. In addition, students will develop the skills and understanding needed for the study of many technical areas build a strong foundation for future study of trigonometric functions of real numbers.
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Duration: Maximum of 9 Days in an A/B schedule

Georgia Standards of Excellence – Mathematics

Content Standards

(Cluster emphasis is indicated by the following icons: Please note that 70% of the time should be focused on the Major Content. ◊ Major Content □ Supporting Content)

Define trigonometric ratios and solve problems involving right triangles

◊ MGSE9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

◊ MGSE9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

◊ MGSE9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice

SMP 1. Make sense of problems and persevere in solving them.

Trigonometry is the study of the relationship between sides and angles of a right triangle and is used to find missing side and angle of the right triangle in real world situations. Students should be able to make sense of the situation by drawing a visual representation and persevere in solving them.

<table>
<thead>
<tr>
<th>Students:</th>
<th>Because Teachers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read the task carefully.</td>
<td>Provide rich tasks aligned to the standards.</td>
</tr>
<tr>
<td>Draw pictures, diagrams, tables, or use objects to make sense of the task.</td>
<td>Allow students time to initiate a plan; uses question prompts as needed to assist students in developing a pathway.</td>
</tr>
<tr>
<td>Discuss the meaning of the task with classmates.</td>
<td>Continually ask students if their plans and solutions make sense.</td>
</tr>
<tr>
<td>Make choices about which solution path to take.</td>
<td>Question students to see connections to previous solution attempts and/or tasks to make</td>
</tr>
<tr>
<td>Try out potential solution paths and make changes as needed.</td>
<td></td>
</tr>
<tr>
<td>Check answers and makes sure</td>
<td></td>
</tr>
</tbody>
</table>
SMP 2. **Reason abstractly and quantitatively.**

Solving right triangles in real world situations involves lengths expressed in different units. Students will have to reason **abstractly and quantitatively** to make sense of quantities and find the relationships to one another in problem situations.

<table>
<thead>
<tr>
<th>Students:</th>
<th>Because Teachers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use mathematical symbols to represent situations</td>
<td>Provide a variety of problems in different contexts that allow students to arrive at a solution in different ways</td>
</tr>
<tr>
<td>Take quantities out of context to work with them (decontextualizing)</td>
<td>Use think aloud strategies as they model problem solving</td>
</tr>
<tr>
<td>Put quantities back in context to see if they make sense (contextualizing)</td>
<td>Attentively listen for strategies students are using to solve problems</td>
</tr>
<tr>
<td>Consider units when determining if the answer makes sense in terms of the situation</td>
<td></td>
</tr>
</tbody>
</table>

SMP 3. **Construct viable arguments and critique the reasoning of others.**

While solving for missing sides/angles of a right triangle, engage students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students should be able to defend their solution or critique the solution of others using estimation of lengths/distance/heights of objects in real situations.

<table>
<thead>
<tr>
<th>Students:</th>
<th>Because Teachers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make and tests conjectures.</td>
<td>Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems, etc.) to support their reasoning.</td>
</tr>
<tr>
<td>Explain and justifies their thinking using words, objects, and drawings.</td>
<td>Question students so they can tell the difference between assumptions and logical</td>
</tr>
<tr>
<td>Listen to the ideas of others and decides if they make sense.</td>
<td></td>
</tr>
<tr>
<td>Ask useful questions.</td>
<td></td>
</tr>
<tr>
<td>Identify flaws in logic when</td>
<td></td>
</tr>
</tbody>
</table>
responding to the arguments of others.

- Elaborate with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence.
- Talks about and asks questions about each other’s thinking, in order to clarify or improve their own mathematical understanding.
- Revise their work based upon the justification and explanations of others.

conjectures.

- Ask questions that require students to justify their solution and their solution pathway.
- Prompt students to respectfully evaluate peer arguments when solutions are shared.
- Ask students to compare and contrast various solution methods.
- Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.).

**SMP 4. Model with mathematics.**

Students should apply the trigonometric concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

<table>
<thead>
<tr>
<th>Students:</th>
<th>Because Teachers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Use mathematical models (i.e. formulas, equations, symbols) to solve problems in the world</td>
<td>- Provide opportunities for students to solve problems in real life contexts</td>
</tr>
<tr>
<td>- Use appropriate tools such as objects, drawings, and tables to create mathematical models</td>
<td>- Identify problem solving contexts connected to student interests</td>
</tr>
<tr>
<td>- Make connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.)</td>
<td></td>
</tr>
<tr>
<td>- Check to see if an answer makes sense within the context of a situation and changing the model as needed</td>
<td></td>
</tr>
</tbody>
</table>
**SMP 5. Use appropriate tools strategically.**

Students may use construction tools like protractor to find the angle of elevation / depression while attempting to find the height of tall structures in their neighborhood as a practical application of the concepts learned. Students might also consider tools that include concrete models to represent a situation, a ruler, a protractor, a compass, a calculator, software, etc.

<table>
<thead>
<tr>
<th><strong>Students:</strong></th>
<th><strong>Because Teachers:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ Use technological tools to explore and deepen understanding of concepts</td>
<td>✓ Make a variety of tools readily accessible to students and allowing them to select appropriate tools for themselves</td>
</tr>
</tbody>
</table>
| ➢ Decide which tool will best help solve the problem. Examples may include:  
  o Calculator  
  o Concrete models  
  o Digital Technology  
  o Pencil/paper  
  o Ruler, compass, protractor | ✓ Help students understand the benefits and limitations of a variety of math tools |
| ➢ Estimate solutions before using a tool | |
| ➢ Compare estimates to solutions to see if the tool was effective | |

**SMP 6. Attend to precision.**

Application of trigonometric concepts involves a lot of calculation that needs to be done with precision. The results also have to be communicated precisely using clear mathematical language.

<table>
<thead>
<tr>
<th><strong>Students:</strong></th>
<th><strong>Because Teachers:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ Communicate precisely using clear language and accurate mathematics vocabulary</td>
<td>✓ Explicitly teach mathematics vocabulary.</td>
</tr>
<tr>
<td>➢ Decide when to estimate or give an exact answer</td>
<td>✓ Provide opportunities for students to check the accuracy of their work.</td>
</tr>
<tr>
<td>➢ Calculate accurately and efficiently, expressing answers with an appropriate degree of precision</td>
<td>✓ Consistently demand and model precision in communication and in mathematical solutions (<em>uses and models correct content terminology</em>).</td>
</tr>
<tr>
<td>➢ Use appropriate units; appropriately labeling diagrams and graphs</td>
<td>✓ Expect students to use precise mathematical vocabulary during mathematical conversations (<em>identifies incomplete responses and asks students to revise their response</em>).</td>
</tr>
</tbody>
</table>

Some of the language used in this document is adapted from the GA Frameworks and Common Core Progressions Documents.
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**Note:** All of the Standards for Mathematical Practice (SMPs) are critical to students fully and appropriately attending to the content. Not all SMPs will occur in every lesson, however SMPs 1, 3, and 6 should be regularly apparent. **All SMPs should be taught in tandem with the content standards.**

**Enduring Understandings**

In order to support deep conceptual learning it is important that students leave this unit experience with the following understandings:

- Similar right triangles produce trigonometric ratios.
- Trigonometric ratios are dependent only on angle measure.
- Trigonometric ratios can be used to solve application problems involving right triangles.
Lesson One Progression

Duration 1-2 Days

Focus Standard(s)

Define trigonometric ratios and solve problems involving right triangles

- MGSE9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Performance Objectives

As a result of their engagement with this unit...

- MGSE-12.G.SRT.6 – SWBAT experiment on ratios of side s of similar right triangles IOT define trigonometric ratios of acute angles in a right triangle.

Building Coherence

Across Grades:

8th Grade - Explain a proof of the Pythagorean Theorem and its converse.
10th Grade - Define trigonometric ratios of acute angles in a right triangle
12th Grade - Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6.

Within Grades:

Explain, using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides

Terms and Definitions

- **Adjacent Side**: In a right triangle, for each acute angle in the interior of the triangle, one ray forming the acute angle contains one of the legs of the triangle and the other ray contains the hypotenuse. This leg on one ray forming the angle is called the adjacent side of the acute angle.
- **Similar triangles**: Triangles are similar if they have the same shape but not necessarily the same size.
  - Triangles whose corresponding angles are congruent are similar.
  - Corresponding sides of similar triangles are all in the same ratio.
Opposite side: In a right triangle, the side of the triangle opposite the vertex of an acute angle is called the opposite side relative to that acute angle.

Properties of Theorems and corollaries:
For the similar triangles as shown above, with angles A, B, and C congruent to angles A’, B’, and C’ respectively, the following proportions follow from the proportion between the triangles.

\[
\frac{a}{w} = \frac{b}{w'} \quad \text{if and only if} \quad \frac{a}{w} = \frac{w}{w'} ;
\]

\[
\frac{a}{w} = \frac{c}{c'} \quad \text{if and only if} \quad \frac{a}{c} = \frac{w}{c'} ;
\]

and

\[
\frac{b}{w'} = \frac{c}{c'} \quad \text{if and only if} \quad \frac{b}{w} = \frac{c}{c'} ;
\]

Three separate equalities are required for these equalities of ratios of side lengths in one triangle to the corresponding ratio of side lengths in the similar triangle because, in general, these are three different ratios. The general statement is that the ratio of the lengths of two sides of a triangle is the same as the ratio of the corresponding sides of any similar triangle.

Thus, for the similar triangles shown above with angles A, B, and C congruent to angles A’, B’, and C’ respectively,

we have that: \( \frac{a}{w} = \frac{b}{w'} = \frac{c}{c'} \)

Trigonometric ratios:
For any acute angle in a right triangle, we denote the measure of the angle by \( \theta \) and define three numbers related to \( \theta \) as follows:

Sine of \( \theta = \sin(\theta) = \frac{a}{c} \)

Cosine of \( \theta = \cos(\theta) = \frac{b}{c} \)

Tangent of \( \theta = \tan(\theta) = \frac{a}{b} \)

Guiding Questions

- What is the Pythagorean Theorem, and when is this theorem used?
- How are the sides and angles of right triangles related to each other?
- How can right triangle relationships be used to solve practical problems?

Interpretations and Reminders

- Students often have difficulties identifying the opposite side and adjacent side of an angle in a right triangle. Students should be exposed to various right triangles with different names and orientation in order to get familiarized with the identification of the sides. Also, students should be encouraged to draw right triangles with any of the three sides including the hypotenuse as base.
- Though it is easy for the teacher to define the three trigonometric ratios and start the lesson, students will benefit more if they complete an investigation on comparing the three ratios using similar right
triangles. The teacher can provide a set of nested similar right triangles for the investigation.

- During the introductory stage, have a poster with a right triangle drawn on the wall as a visual tool. Color-code the definitions by drawing the opposite side in green and writing the word ‘opposite’ in green, drawing the adjacent side in red and writing the word ‘adjacent’ in red; drawing the hypotenuse in blue and writing the word ‘hypotenuse’ in blue and so on.

### Misconceptions

- Some students assume that right triangles must be oriented a certain way. Expose them to various right triangles with different orientations.
- Students often assume that the vertical leg of a right triangle is always the opposite side irrespective of what the angle is. Let students know that the opposite side is dependent on the angle considered.
- Students assume that when the length of the sides of the right triangle changes with angle remaining the same, the ratio of two sides changes. Students need to discover that even when the lengths of the sides of the individual triangles grow or shrink, the ratio remains the same. Only changes in the angle affect the ratio.
- Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.

### Suggested Learning Experiences

**Procedural Fluency:** (Recommended for 5 - 10 minutes each day: Fluency strategies are useful to activate student voice, solicit prior knowledge and develop fluency based on conceptual understandings.) For additional fluency practice strategies see the table at the end of this document.

While introducing trigonometric ratios for the first time, it is worthwhile to spend a few minutes on identifying the opposite and adjacent sides of an angle in a right triangle. Also, students should be exposed to right triangles drawn with different orientation.

Given triangle ABC, right angled at C,

a) What is the side opposite to ∠A?
b) What is the side adjacent to ∠A?
c) What is the side opposite to ∠B?
d) What is the side adjacent to ∠B?

Given triangle ABC, right angled at F,

a) What is the side opposite to ∠D?
b) What is the side adjacent to ∠D?
c) What is the side opposite to ∠E?
d) What is the side adjacent to ∠E?

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Given triangle PQR, right angled at R,
   a) What is the side opposite to ∠P?
   b) What is the side adjacent to ∠P?
   c) What is the side opposite to ∠Q?
   d) What is the side adjacent to ∠Q?

Note: The above activity can also be done as an oral activity where the teacher marks and angle and points to each side of the triangle while students give a choral response as opposite, adjacent, and hypotenuse.

**Graduated Measure** (The graduated measure is a quick opportunity to diagnose students’ level of comfort with the material before you begin the progression.)

Student Directions: Select a question/problem from the table below that you feel best equipped to answer successfully.

<table>
<thead>
<tr>
<th>Beginning</th>
<th>Developing</th>
<th>Proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for x:</td>
<td>Solve for NM and NL given triangle KJH is similar to triangle NML:</td>
<td>Solve for x:</td>
</tr>
<tr>
<td>![Diagram of triangle PQR]</td>
<td>![Diagram of similar triangles]</td>
<td>![Diagram of triangle RST]</td>
</tr>
</tbody>
</table>

**Gradual Release of Responsibility**

The standard in this lesson progression on understanding that the side ratios of a right triangle are dependent on the angles and not necessarily the lengths of the sides has to be investigated by students as an inquiry based lesson for better understanding of the concept. As per the gradual release model for inquiry-based lesson, students will start with a collaborative activity and then will discuss their findings with the class under teacher’s supervision and finally the teacher can summarize the concept.
Collaborative Practice:
Introduction to Trigonometric Ratios
Given below is a set of 4 right triangles that are similar to each other.
1. Name the four right triangles in the above figure.

2. Explain why the four right triangles are similar to each other.

3. Use a ruler to measure each side of the four triangles and complete the table below.

Note: When completing the table for the sides, name the side before giving the length measure. For example, in $\triangle APQ$, the side adjacent to $\angle A$ will be $AP = \underline{\quad}$ cm.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Side opposite to $\angle A$</th>
<th>Side adjacent to $\angle A$</th>
<th>Hypotenuse $\frac{O}{H}$ (Round to 2 decimal places)</th>
<th>$\frac{A}{H}$ (Round to 2 decimal places)</th>
<th>$\frac{C}{A}$ (Round to 2 decimal places)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APQ</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ARS</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Based on your table, answer the questions below:

4. What do you observe from the above table?

5. Are the side lengths of corresponding sides of any two triangles the same?

6. What do you observe about the ratio \( \frac{c}{h} \) for all the four triangles? Write in your own words what this means.

7. What do you observe about the ratio \( \frac{a}{h} \) for all the four triangles? Write in your own words what this means.

8. What do you observe about the ratio \( \frac{c}{a} \) for all the four triangles? Write in your own words what this means.

**Guided Practice:** Once students are completed with the above activity, teacher can initiate a discussion amongst students by asking them to share their response. The teacher can finally summarize the concept and clarify misconceptions based on what students have written and discovered. The following questions can be used to strengthen their conceptual understanding:

a) Say \( m \angle A = 25^0 \), in the above figure. If we change the angle measure of \( A \) to (say) \( 52^0 \), what all changes in the table?

b) What happens to each of the three ratios?

c) Will the three ratios be the same as what you have now?

d) If not, will the ratios be the same for all the four triangles?

e) If I draw another right triangle JKL right angled at \( K \) and \( m \angle J = 25^0 \), what do you know about the three ratios \( \frac{c}{h}, \frac{a}{h}, \frac{c}{a} \) for angle \( J \)?

f) Complete the statement based on the above observations and discussion:
**Discovery Statement:** In a right triangle, the three ratios, namely $\frac{G}{H}$, $\frac{A}{S}$, and $\frac{G}{A}$ for any given acute angle is not dependent on the ________________ of the triangle and only depends on the measure of the ________________ of the triangle.

**Note to the teacher:** A similar inquiry based activity for discovering that the ratios are a constant from NY Engage is provided here. You can see the student version and the teacher version.

[Trig ratios Intro-NY engage(Student).pdf](Trig ratios Intro-NY engage(Student).pdf)  [Trig ratios Intro-NY engage(Teacher).pdf](Trig ratios Intro-NY engage(Teacher).pdf)

**Focus Lesson:** Using the above examples and more teacher created examples, teacher will define the three trigonometric ratios of any given acute angle using a right triangle.

For any acute angle in a right triangle, we denote the measure of the angle by $\theta$ and define three numbers related to $\theta$ as follows:

<table>
<thead>
<tr>
<th>Sine of $\theta = \sin(\theta)$</th>
<th>$= \frac{\text{Opposite Side}}{\text{Hypotenuse}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos(\theta) = \cos(\theta)$</td>
<td>$= \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$</td>
</tr>
<tr>
<td>$\tan(\theta) = \tan(\theta)$</td>
<td>$= \frac{\text{Opposite Side}}{\text{Adjacent Side}}$</td>
</tr>
</tbody>
</table>

All right triangles with a given acute angle say ($\theta$) are similar. For similar triangles, the ratio of the lengths of two particular sides in one triangle is the same as the ratio of the lengths of the corresponding sides in any other similar right triangle (this is what students proved above). The ratio depends only on the angle measure of the acute angle under consideration and not on the individual sides of the triangle. For every angle measure $\theta$ between $0^0$ and $90^0$, there is a unique value for $\frac{G}{H}$ (also defined as Sine $\theta$ or sin $\theta$ in short), a unique value for $\frac{A}{S}$—(also defined as Cosine $\theta$ or cos $\theta$ in short), and an unique value for $\frac{G}{A}$—(also defined as tangent $\theta$ or tan $\theta$ in short).
Once the trigonometric ratios have been introduced, students are required to memorize and remember the definitions and what better way to remember than the use of mnemonics?

\[ \text{SOH-CAH-TOA} \]

\[ \text{sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, and tangent equals opposite over adjacent.} \]

Encourage students to create their own mnemonics in order to retain the concept learned.
In the given right triangle, find the value of the following:

a) $\sin x$  b) $\sin W$  c) $\cos X$  d) $\cos W$
e) $\tan X$  f) $\tan W$

In the given right triangle, find the value of the following:

a) $\sin A$  b) $\sin B$  c) $\cos A$  d) $\cos B$
e) $\tan A$  f) $\tan B$

Use the 30-60-90 triangle to find the trigonometric ratios of $30^0$ and $60^0$.

Use the 45-45-90 triangle to find the trigonometric ratios of $45^0$.

(Focus Lesson) Trigonometric ratios using calculators: Now that students have learned to evaluate the trigonometric ratios from a right triangle and that students also know that the ratios of any acute angle does not depend on the sides of the triangle, but only depends on the angle, teacher will have to initiate a discussion to evaluate the trigonometric ratios of any acute angle.

Simone constructs a right triangle $ABC$ with $\angle A = 38^0$ and $\angle B = 90^0$ and $\angle C = 52^0$. Simone calculates the three trigonometric ratios of $\angle A$ as $\sin A = x$, $\cos A = y$ and $\tan A = z$. 
Jasmine constructs another right triangle PQR with \( \angle P = 38^0 \) and \( \angle Q = 90^0 \) and \( \angle R = 52^0 \). Jasmine calculates the three trigonometric ratios of \( \angle P \) as Sin \( P = d \), Cos \( P = e \) and tan \( P = f \).

What do you know about the two sets of ratios?

Students should be able to reason out that \( x=d \), \( y=e \), \( z=f \) and justify based on their investigation on similar right triangles done in the beginning of this unit. Teacher can use this as Segway to inform how mathematicians had calculated the trigonometric ratio for every acute angle with precision and how it was presented, as a table of values earlier and then has been made available in the calculators.

Help students read the trigonometric values from a calculator and also help them to make sure that the calculator mode is set to degree for the current unit.

Though inverse trigonometric functions are not part of the standards in this unit, students should be taught to find a given angle given one of the three ratios. Help students evaluate the angle measure given any trigonometric function for the angle.

- What is Sin 38^0?
- Find the value of tan 24.6^0
- Given Cos B = 0.345, what is B?
- Given sinP = 0.75, find P.
- What is the value of A if SinA = cos A
- Can Sin A be greater than 1? Why or why not?

### Additional Unit Assessments

<table>
<thead>
<tr>
<th>Assessment Name</th>
<th>Assessment</th>
<th>Assessment Type</th>
<th>Standards Addressed</th>
<th>Cognitive Rigor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discovering Special triangles</td>
<td>Discovering Special Triangles.docx</td>
<td>Learning Task</td>
<td>MGSE9-12.G.SRT.6</td>
<td>DOK 3</td>
</tr>
</tbody>
</table>

### Differentiated Supports

- **Learning Difficulty**
  - Challenge gifted students to find the range of the three trigonometric functions for

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<table>
<thead>
<tr>
<th><strong>High Achieving</strong></th>
<th>any acute angle. Ask them to justify why sine and cosine functions cannot have a value greater than 1.</th>
</tr>
</thead>
</table>
| **English as a Second Language** | • Have students write their understandings in a math journal. You may want to consider allowing EL students to write notes in their first language and annotate by identifying math specific words they do not have a translation for.  
• Allow students to create pictorial flash cards for new math words.  
• Create a graphic organizer that outlines the relationships in each special right triangle. |
| **Online/Print Resources** | **Digital Resources**  
• [https://learnzillion.com/lesson_plans/709#lesson](https://learnzillion.com/lesson_plans/709#lesson)  
• [http://www.mathwarehouse.com/geometry/](http://www.mathwarehouse.com/geometry/)  
• [https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles](https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles)  
• [https://www.teachingchannel.org/videos/introduction-to-trigonometry](https://www.teachingchannel.org/videos/introduction-to-trigonometry)  
| **Print Resources** | [Glencoe -Trig Ratios Practice.pdf](Glencoe-Practice.pdf) |
| **Manipulatives and Tools** | | |
| **Textbook Alignment** | **Geometry, McGraw-Hill**  
pp. 558 - 578 |
Lesson Two Progression

**Duration 2-3 Days**

### Focus Standard(s)

- **MGSE9-12.G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

### Performance Objectives

**As a result of their engagement with this unit...**

- **MGSE9-12.G.SRT.7** – SWBAT verify the relationship between the sine and cosine of complementary angles, IOT apply the same in solving right triangles.

### Building Coherence

#### Across Grades:

- **7th Grade**
  - Solve problems involving supplementary and complementary angles

- **10th Grade**
  - Relationship between the sine and cosine of complementary angles

- **12th Grade**
  - Relationship between sine and cosine, secant and cosecant and tan and cotangent of complementary angles

#### Within Grades:

Define trigonometric ratios, find relationship between sine and cosine of complementary angles and use the same to solve for missing sides and missing angles of right triangles

### Terms and Definitions

**Note:** A number of these definitions have been introduced in lesson progression 1, but are given here again as they will be repeatedly used in this lesson also.

<table>
<thead>
<tr>
<th><strong>Adjacent Side:</strong></th>
<th><strong>Theorem on Complementary Angles:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>In a right triangle, for each acute angle in the interior of the triangle, one ray forming the acute angle contains one of the legs of the triangle and the other ray contains the hypotenuse. This leg on one ray forming the angle is called the adjacent side of the acute angle.</td>
<td>For each pair of complementary angles in a right triangle, the sine of one angle is the cosine of its complement. This means, if A and B are the acute angles of a right triangle ABC, then...</td>
</tr>
</tbody>
</table>
**Complementary angles:** Two angles whose sum is 90° are called **complementary.** Each angle is called the complement of the other.

**Opposite side:** In a right triangle, the side of the triangle opposite the vertex of an acute angle is called the opposite side relative to that acute angle.

**Similar triangles:** Triangles are similar if they have the same shape but not necessarily the same size.

Triangles whose corresponding angles are congruent are similar.

Corresponding sides of similar triangles are all in the same proportion.

Thus, for the similar triangles shown above

with angles A, B, and C congruent to angles A', B', and C' respectively,

we have that: \( \frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} \)

\[ \text{Sin A} = \text{Cos B} \quad \text{and} \quad \text{Sin B} = \text{Cos A} \]

**Trigonometric ratios:**

For any acute angle in a right triangle, we denote the measure of the angle by \( \theta \) and define three numbers related to \( \theta \) as follows:

\[ \text{Sine of } \theta = \sin(\theta) = \frac{l}{h} \]

\[ \text{Cosine of } \theta = \cos(\theta) = \frac{a}{h} \]

\[ \text{Tangent of } \theta = \tan(\theta) = \frac{a}{h} \]

**Guiding Questions**

- How are the sine and cosine function of the two acute angles in a right triangle related to each other?

**Interpretations and Reminders**

- Remind students that in a right-angled triangle, the two acute angles are complementary as their sum is 90°. This is true only for right-angled triangles.

- Complementary angles can be found only for angles less than 90° or acute angles. The complement of an angle cannot be negative.
• There is only one angle (45°) whose complement is itself.

**Misconceptions**

• Students often mistake complementary and supplementary angles. Two angles are called supplementary if the sum of the two angles is 180°. Two angles are called complementary if the sum of the two angles is 90°.

• Some students think any three angles whose sum is 90° as complementary angles and that supplementary angles may be any number of angles whose sum is 180°. Help them understand that complementary and supplementary angles refer only to a pair of angles.

**Suggested Learning Experiences**

**Procedural Fluency:** (Recommended for 5 - 10 minutes each day: Fluency strategies are useful to activate student voice, solicit prior knowledge and develop fluency based on conceptual understandings.) For additional fluency practice strategies see the table at the end of this document.

![Fluency](image)

By this time, students should be familiar with writing trigonometric ratios for a given angle from a right-angled triangle and also be able to draw a right-angled triangle given a trigonometric ratio. Since the trigonometric ratio of an acute angle does not necessarily depend on the individual sides of the triangle, it is possible to draw any number of similar triangles to represent a given ratio.

Given sin A = 0.3, draw at least 3 triangles to represent the situation and find the length of each side.

Example Solution: Given SinA = 0.3 = \( \frac{3}{10} = \frac{6}{20} = \frac{9}{30} \)

![Diagram](image)

**Graduated Measure** (The graduated measure is a quick opportunity to diagnose students’ level of comfort with the material before you begin the progression.)

Student Directions: Select a question/problem from the table below that you feel best equipped to answer successfully.
Gradual Release of Responsibility

In this lesson progression again, the standard demands that students complete an investigation (an inquiry based lesson) for better understanding of the concept. As per the gradual release model for inquiry-based lesson, students will start with a collaborative activity and then will discuss their findings with the class under teacher’s supervision and finally the teacher can summarize the concept.

Collaborative Practice:

A and B are the acute angles of a right-angled triangle. For each of the triangles given below, calculate the value of Sin A, Cos B, Sin B and Cos A.

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Sin A</th>
<th>Cos B</th>
<th>Sin B</th>
<th>Cos A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What do you know about angles A and B in terms of their angle measure? (Complementary)
2. What do you observe from the table?

Use a calculator to complete the table given below:

<table>
<thead>
<tr>
<th>Angle A</th>
<th>B, the complement of A</th>
<th>SinA</th>
<th>CosB</th>
<th>SinB</th>
<th>CosA</th>
</tr>
</thead>
<tbody>
<tr>
<td>25°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What do you observe from the table?

Focus Lesson:

The teacher will initiate a discussion on the above results and help students write their observation in the form of a mathematical statement as follows: The sine of any acute angle is equal to the cosine of its complement and the cosine of any acute angle is equal to the sine of its complement.

The above statement can also be written as $\sin \theta = \cos(90^\circ - \theta)$ and $\cos \theta = \sin(90^\circ - \theta)$

In a right triangle PQR right angled at Q, if $\sin R = \sin P$, what do you know about P?

Given $\sin 35^\circ = \cos x$, what is the value of x?

Given $\sin A = 0.5$, what is $\cos (90^\circ - A)$?

Given $\cos P = \frac{1}{2}$, what is $\sin(90^\circ - P)$?

What is $\sin P$?
**Independent Practice:**
Worksheet on sine and cosine of complementary angles.

**Additional Unit Assessments**

<table>
<thead>
<tr>
<th>Assessment Name</th>
<th>Assessment Type</th>
<th>Standards Addressed</th>
<th>Cognitive Rigor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine and Cosine of Complementary Angles (EngageNY)</td>
<td>Discovery Task</td>
<td>MGSE9-12.G.SRT 7</td>
<td>DOK 2</td>
</tr>
<tr>
<td>Trigonometric Function Values</td>
<td>Learning Task</td>
<td>MGSE9-12.G.SRT 7</td>
<td>DOK 2/3</td>
</tr>
</tbody>
</table>

**Differentiated Supports**

**Learning Difficulty**

In order for students to understand the relationship between sine and cosine of complementary angles, students will need to see more examples of trig ratios of complementary angle and then compare and discover for themselves. Provide as many examples as need until they are able to discover the relationship.

**High Achieving**

Challenge high achieving students with the following extension task.

**English as a Second Language**

- Have students write their understandings in a math journal. You may want to consider allowing EL students to write notes in their first language and annotate by identifying math specific words they do not have a translation for.
- Allow students to create pictorial flash cards for new math words.
- Create a graphic organizer that outlines the relationships between the Sine and Cosine of complementary angles.
### Online/Print Resources

<table>
<thead>
<tr>
<th>Digital Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <a href="http://mathbitsnotebook.com/Geometry/Trigonometry/TGTrigSineCosine.html">http://mathbitsnotebook.com/Geometry/Trigonometry/TGTrigSineCosine.html</a></td>
</tr>
<tr>
<td>• <a href="https://www.opened.com/homework/g-srt-7-explain-and-use-the-relationship-between-the-sine-and/3692317">https://www.opened.com/homework/g-srt-7-explain-and-use-the-relationship-between-the-sine-and/3692317</a></td>
</tr>
</tbody>
</table>

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<th>Print Resources</th>
</tr>
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<tbody>
<tr>
<td><a href="#">NY Engage Sine, Cos of comp angles.pdf</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td><a href="#">Compass</a></td>
</tr>
<tr>
<td><a href="#">Protractor</a></td>
</tr>
<tr>
<td><a href="#">Ruler</a></td>
</tr>
<tr>
<td><a href="#">Sketchpad</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Textbook Alignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry, McGraw-Hill</td>
</tr>
</tbody>
</table>
### Lesson Three Progression

**Duration 2-3 Days**

## Focus Standard(s)

1. ** MGSE9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**

## Performance Objectives

**As a result of their engagement with this unit...**

- ** MGSE9-12.G.SRT.8 – SWBAT apply trigonometric ratios and Pythagorean Theorem IOT solve for unknown angles and unknown sides of right triangles in real life situations.**

## Building Coherence

### Across Grades:

- **8th Grade**
  - Explain a proof of the Pythagorean Theorem and its converse.

- **10th Grade**
  - Application of trigonometric ratios to find height of trees/buildings, width of river, etc. (real world applications)

- **12th Grade**
  - Application of law of sines, cosine formula to find height of trees/buildings, width of river, etc. (real world applications in all triangles)

### Within Grades:

**Define trigonometric ratios, find relationship between sine and cosine of complementary angles and use the same to solve for missing sides and missing angles of right triangles**

## Terms and Definitions

### Angle of Depression:

The angle below horizontal that an observer must look to see an object that is lower than the observer. Note: The angle of depression is congruent to the angle of elevation (this assumes the object is close enough to the observer so that the horizontals for the observer and the object are effectively parallel; this would not be the case for an astronaut in orbit around the earth observing an object on the ground).

### Angle of Elevation:

The angle above horizontal that an observer must look to see an object that is higher than the observer. Note: The angle of elevation is congruent to the angle of depression (this assumes the object is close enough to the observer so that the horizontals for the observer and the object are effectively parallel; this would not be the case for a ground tracking station observing a satellite in orbit around the earth).
Guiding Questions

- How can right triangle relationships be used to solve practical problems?

Interpretations and Reminders

- In this lesson, students will be using trigonometric functional values of various angles to solve for missing side of a right triangle. Remind students that sine and cosine values have always got to be less than 1 for any acute angle.
- Drawing a model to find unknown sides/angles and marking known sides/angles on the figure helps in organizing thoughts to solve any right triangle problem.
- While solving for one of the sides of a right triangle, it is worthwhile to note that the hypotenuse is always the longest side and the side opposite to the smaller angle is shorter than the side opposite to the larger angle. This fact can be used to eliminate wrong choices while working on multiple-choice questions and also to check the solution after solving.
- In order for students to gain mastery in this standard, students should be exposed to a variety of application problems. The essence lies in drawing a model to represent the situation after which solving for the unknown is an easy process.
- Encourage students to create their own word problems using information around them.

Misconceptions

- Students sometimes mark the angle between the vertical line and the line of sight as the angle of depression instead of the angle between the line of sight and the horizontal while looking down at an object. Remind students that we always see things that lie on a horizontal line with our eyes.
- When doing problems involving angle of elevation, students sometimes forget to add the height of the person involved. The height is not a small quantity to be ignored.
- When calculating the missing side of a right triangle, the trigonometric ratio has to be entered to 4 decimal places and after appropriate step can be rounded to the requirement in the question. The rounding cannot happen in the first step unless specified.
Suggested Learning Experiences

**Procedural Fluency:** (Recommended for 5 - 10 minutes each day: Fluency strategies are useful to activate student voice, solicit prior knowledge and develop fluency based on conceptual understandings.) For additional fluency practice strategies see the table at the end of this document.

By this time, students should be familiar with use of calculators to find trigonometric ratios. Ask students to find the following:

1. Sin 29.45°
2. Cos 76°
3. Tan 34°
4. Sin A = 0.76, find A,
5. Cos B =0.66, find B
6. Tan P = 3.45, find P.

**Graduated Measure** (The graduated measure is a quick opportunity to diagnose students’ level of comfort with the material before you begin the progression.)

Student Directions: Select a question/problem from the table below that you feel best equipped to answer successfully.

<table>
<thead>
<tr>
<th>Beginning</th>
<th>Developing</th>
<th>Proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

**Gradual Release of Responsibility**

**Focus Lesson:** Students will not be learning any new concept in this lesson but will be applying the concepts learned. Teacher can first introduce the vocabulary words ‘angle of elevation and angle of depression’ and model application problems.

**Example 1:**
Solve for x and y in the given right triangle

\[
\cos 20 = \frac{a}{ny} = \frac{1}{y}, \quad 0.4080 = \frac{1}{y}, \quad 0.4080y = 20, \quad y = \frac{1}{0.4080} = 29.41
\]

\[
\tan 20 = \frac{a}{ny} = \frac{x}{y}, \quad 2.2371 = \frac{x}{y}, \quad 20(2.2371) = x, \quad x = 44.742
\]
Note: After finding $y$, $x$ could have been found using the sine function as well. However, an error in finding $y$ will lead to another error in finding $x$. Hence it is better to find the second variable using given quantities if possible.

**Example 2:**

2. The main character in a play is delivering a monologue, and the lighting technician needs to shine a spotlight onto the actor’s face. The light being directed is attached to a ceiling that is 10 feet above the actor’s face. When the spotlight is positioned so that it shines on the actor’s face, the light beam makes an angle of $20^\circ$ with a vertical line down from the spotlight. How far is it from the spotlight to the actor’s face? How much further away would the actor be if the spotlight beam made an angle of $32^\circ$ with the vertical?

**Comment(s):**

Students are limited to sine and cosine functions, so the equation to be solved here requires that the unknown be in the denominator. Students should be prompted to use what they know about solving rational equations, that is, to multiply by the LCD first to eliminate the rational expressions. Teachers can encourage students to leave the trigonometric expression in the equation until they are ready to calculate, as shown in the solution, so that they use all the accuracy of the calculator.

**Solution(s):**

We are given the length of the side adjacent to the $20^\circ$ angle and asked to find the length of the hypotenuse. The cosine ratio is adjacent/hypotenuse.

Let $x$ be the unknown distance from the spotlight to the actor’s face. Then,

$$\cos(20^\circ) = \frac{10 \text{ ft}}{x} \rightarrow x \cdot \cos(20^\circ) = 10 \text{ ft} \rightarrow x = \frac{10 \text{ ft}}{\cos(20^\circ)} \approx 10.64 \text{ ft}$$

For an angle of $32^\circ$, we have:

$$\cos(32^\circ) = \frac{10 \text{ ft}}{x} \rightarrow x \cdot \cos(32^\circ) = 10 \text{ ft} \rightarrow x = \frac{10 \text{ ft}}{\cos(32^\circ)} \approx 11.79 \text{ ft}$$

So, the actor’s face is approximately $11.79 - 10.64 = 1.15$ feet further away with the larger angle.

*The above problem is an example from GSE Frameworks-Unit 3*
Example 3:

**To measure the width of a river.** Two trees stand opposite one another, at points A and B, on opposite banks of a river.

Distance AC along one bank is perpendicular to BA, and is measured to be 100 feet. Angle ACB is measured to be 79°. How far apart are the trees; that is, what is the width \( w \) of the river?

From the right triangle ABC, \( \tan C = \tan 79^\circ = \frac{w}{1} \)

\[ W = 100 \tan 79^\circ = 49.57 \]

The above problem is an example from ‘The math page’.

Guided Practice:

6. An observer in a lighthouse sees a sailboat out at sea. The angle of depression from the observer to the sailboat is 6°. The base of the lighthouse is 50 feet above sea level and the observer’s viewing level is 84 feet above the base. (See the figure at the right, which is not to scale.)

**Comment(s):**

To be successful in solving the parts of this item, students must realize that the two legs of the right triangles to be used lie on the horizontal line at sea level and the vertical line through the observer’s position. Figures provided with the solutions show these right triangles.

What’s the distance from the sailboat to the observer?
Solution(s):

If we extend the line from the observer to the base of the lighthouse down to sea level, we have a right triangle containing a $6^\circ$ angle. The side opposite the $6^\circ$ angle has length 134 feet and the length, $x$, of the hypotenuse is the distance from the sailboat to the observer.

$$\sin(6^\circ) = \frac{134'}{x}$$

$$x = \frac{134'}{\sin(6^\circ)} \approx 1282'$$

Thus, the (straight line) distance from the sailboat to the observer is approximately 1282 feet.

The above problem is an example from GSE Frameworks-Unit 3

Note: The teacher needs to constantly do formative assessments and model as many example problems as needed by her/his students before releasing them to work collaboratively and independently.

Formative Assessment

1. Check to see if students are able to create a visual model (diagram) to represent a real world situation using a right triangle and mark appropriate quantities.
2. Check to see if students are able to set up a ratio to solve for a missing side or angle in application problems.

Examples: Draw a visual model to represent the following situations:

1. A ladder is leaning against a building. The ladder reaches a height of 20 feet on the building. The ladder makes an angle of $28^\circ$ with the wall.
2. The main character in a play is playing a solo, and the lighting technician needs to shine a spotlight into the actor’s face. The light being directed is attached to the ceiling that is 12 feet above the actor’s face. The angle of depression to the actor’s face is $18^\circ$.
3. A tree casts a 60 foot shadow. The angle of elevation is $30^\circ$. This is the angle at which you look up to the top of the tree from the ground. What is the trigonometric ratio that needs to be used to find the height of the tree?
What trigonometric ratio can be used to find the height of the tree?

Collaborative Practice: This activity given below has been taken from NCTM. This activity helps students to internalize how real world applications can be interpreted and how to reason real world situations abstractly and quantitatively.

Noteworthy Commentary from the National Council of Teachers of Mathematics (NCTM):

Encourage students to use a calculator. Students should then work in small groups, comparing ideas. As students begin work, emphasize that many "correct" answers are possible but that teams should agree on one answer to each problem. Finally, initiate a whole-class discussion; record the consensus of answers to 1(a), 1(b), and 1(c) on the overhead transparency; and discuss the different groups' answers to problem 2.

- 1(a) Answers will vary. Some may say that Chris is standing at the base of the cliff and looking straight up; others may say that he could be back infinitely far. Others may argue that the distance would have to be at least a few feet from the base to actually identify the top. They may also say that limited ability to see will require that Chris be no more than a few miles away.

- 1(b) The greater the distance, the greater the height.

- 1(c) The greater the angle, the greater the height. 1(c) Answers will vary. Many students will simply swap angle measures.

- 2(b) The angle measures must be 60 degrees, but distance will vary.

- 2(c) The sum of the selected angle measures must be 90 degrees, but distance will vary.

- 2(d) One of the selected measures must be 90 degrees. Other answers will vary.
Collaborative Activity: **Clinometer Activity**

In order for students to make a deeper conceptual understanding of angle of elevation/angle of depression, students should be provided an opportunity to measure tall structures within the school building using a clinometer. A clinometer can be easily constructed using a protractor and a straw or students can also use a clinometer app on their cell phones. Ask students to work in groups of 3, collect data (angle of elevation/depression, horizontal distance) by actual measurements and then calculate the height (vertical) that they intended to calculate. Use

![Clinometer Diagram](https://example.com/Clinometers_Angles_of_Elevations_and_Trigonometry.pdf)

**Independent Practice:**

![Schmoop-SRT 8 worksheet w answers](https://example.com/Schmoop-SRT_8.pdf)

**Additional Unit Assessments**

<table>
<thead>
<tr>
<th>Assessment Name</th>
<th>Assessment</th>
<th>Assessment Type</th>
<th>Standards Addressed</th>
<th>Cognitive Rigor</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRT 8 Assessment</td>
<td><img src="https://example.com/SRT_8_Assessment.pdf" alt="SRT 8 Assessment" /></td>
<td>Formative Assessment</td>
<td>MGSE9-12.G.SRT.8</td>
<td>DOK3</td>
</tr>
<tr>
<td><img src="https://example.com/SRT_8_Assessment_Rubric.pdf" alt="SRT 8 Assessment Rubric" /></td>
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</tr>
<tr>
<td>Constructing Special Angles</td>
<td><img src="https://example.com/Constructing_Special_Angles.pdf" alt="Constructing Special Angles" /></td>
<td>Learning Task</td>
<td>MGSE9-12.G.SRT.8</td>
<td>DOK3</td>
</tr>
<tr>
<td>Ask the Pilot</td>
<td><img src="https://example.com/Ask_the_Pilot.pdf" alt="Ask the Pilot" /></td>
<td>Individual/Partner Task</td>
<td>MGSE9-12.G.SRT.8</td>
<td>DOK3</td>
</tr>
</tbody>
</table>

Some of the language used in this document is adapted from the GA Frameworks and Common Core Progressions Documents.
### Differentiated Supports

**Learning Difficulty**
For students with learning difficulty, it might take some time to represent a word problem as a right triangle model. Give written step-by-step instructions that they can follow. Also, provide visuals for word problems initially until they are able to draw models on their own.

![Solving right triangles worksheet](image)

**High Achieving**

![PDF](image)

Glencoe-SRT 8Trig Enrichment activity

**English as a Second Language**
- Have students write their understandings in a math journal. You may want to consider allowing EL students to write notes in their first language and annotate by identifying math specific words they do not have a translation for.
- Allow students to create pictorial flash cards for new math words.
- Create a graphic organizer that outlines the problem solving process for solving contextual problems with trigonometry.

### Online/Print Resources

**Digital Resources**
- [https://prezi.com/cswqjoa8s1y/right-triangle-trigonometry-activity/](https://prezi.com/cswqjoa8s1y/right-triangle-trigonometry-activity/)
- [http://www.themathpage.com/atrig/solve-right-triangles.htm](http://www.themathpage.com/atrig/solve-right-triangles.htm)

**Print Resources**
- NY Engage SRT 8 Teacher.docx
- NY Engage SRT 8 Student.docx

**Manipulatives and Tools**
- Compass
- Protractor
- Ruler
- Sketchpad

**Textbook Alignment**
<table>
<thead>
<tr>
<th><strong>Sample Fluency Strategies for High School</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expression of the Week</strong></td>
</tr>
<tr>
<td><strong>Algebraic Talks</strong></td>
</tr>
</tbody>
</table>
| **Algebraic Talks (The Norms):** | - No Pencils  
- No calculators  
- Mental Math Only  
- Consider the problem individually during the first few minutes |
| | Place a problem on the board and allow students to consider possible solution pathways. Next, allow 5 or 6 student to explain their strategy and record the various methods on the board for all students to see. Name the strategy (preferably after the student who made the conjecture). Allow other students to ask questions about the strategies while the proposing student is allowed to defend his/her work. |
| | Also see: Jo Boaler Number Talks Videos |
| **Picture This** | Provide students with a graph, or chart and ask them to come up with a contextual situation that makes the picture relevant |
Find the odd man out

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td>( x - y = 5 )</td>
<td>( 2x + 3y = 6 )</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td><strong>D</strong></td>
</tr>
<tr>
<td>( 2x - 3y = 0 )</td>
<td>( 3y = 2x - 4 )</td>
</tr>
</tbody>
</table>

**Note 1:** This is an open-ended question where students can choose any one of the choices A-D as the odd man as long as they are able to defend their choice. For example, 
A is the odd equation because it is in the only equation with coefficients as 1 or -1  
B is the odd equation because it is in the only equation with negative slope  
C is the odd equation because it is in the only equation whose line passes through the origin  
D is the odd equation because it is in the only equation which is not represented in the standard form.

**Note 2:** Teacher can provide 4 equations/ 4 triangles/ 4 transformations/ 4 trig ratios where there is more than one way of classifying. Students can choose the odd man by mere inspection (surface level) or by examining the four components with a deeper understanding of the concept.

**Example 2:**
A is the odd equation because it has all positive coefficients or because it has two negative roots.

B is the odd equation because it has two positive roots.

C is the odd equation because it has one positive and one negative root.

D is the odd equation because the parabola opens downwards.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2 + 5x + 6$</td>
<td>$y = x^2 - 5x + 6$</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>$y = x^2 - x - 6$</td>
<td>$y = -x^2 + 6$</td>
</tr>
</tbody>
</table>

Example 1: Draw as many triangles as possible that are similar to

Note: Teachers can ask students to generate examples and non-examples for a variety of topics to improve fluency. A few more possibilities are listed here:
1. Create quadratic equations with imaginary roots
2. Create quadratic equations with complex roots
3. Create non-examples of complementary angles.
4. Create examples of circles whose center is (3, -2)
5. Draw triangle ABC where Sin A = 0.3

Choose a student's work to be displayed on board after removing the name and ask the class to locate the error and also work the solution correctly. This can be done in any topic where the problem requires multiple steps to solve.